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Predicting Corporate Financial Distress: A Time-Series CUSUM Methodology

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Abstract. The ability to predict corporate financial distress can be strengthened using models that account for serial correlation in the data, incorporate information from more than one period and include stationary explanatory variables. This paper develops a stationary financial distress model for AMEX and NYSE manufacturing and retailing firms based on the statistical methodology of time-series Cumulative Sums (CUSUM). The model has the ability to distinguish between changes in the financial variables of a firm that are the result of serial correlation and changes that are the result of permanent shifts in the mean structure of the variables due to financial distress. Tests performed show that the model is robust over time and outperforms similar models based on the popular statistical methods of Linear Discriminant Analysis and Logit.

Key words: financial distress models, linear discriminant analysis logit model, non-stationary financial ratios, time-series CUSUM, vector autoregressive process

JEL Classification: C32, G33, M41

1. Introduction

Financial variables included in financial distress models as explanatory variables generally exhibit positive serial correlation over time, e.g., Theodossiou (1993).[1] As such, positive deviations in these variables from their long-run equilibrium means in one period are followed by positive deviations in subsequent periods while negative deviations are followed by negative deviations. The magnitude of these deviations depend on the degree of serial correlation inherent in the financial variables as well as a random white noise error term. The presence of serial correlation may be attributed to active attempts by the management to align the variables with their population means and/or systematic micro- and macroeconomics effects operating on the firm, e.g., Lee and Wu (1988). The random error term may be attributed to firm-specific or economic random shocks.

Under stationarity of the time-series process, the variables have a tendency to revert around their long-run equilibrium means over time. The latter implies that the deviations of the variables from their means are transitory over time. Thus, the variables have a tendency to return back to their mean values in the near future. The time needed for the variables for such a return (i.e., the persistence of deviations) depends on the degree of serial correlation inherent in the variables.[2]

In addition to the serial correlation and white noise error components, both of which are transitory, important variables for financially distressed firms include a non-transitory component which is due to permanent shifts in the mean structure of the variables toward the failed population. These shifts are initially small in magnitude and become larger as the firms approach the point of economic collapse. Past financial distress models based on Linear Discriminant Analysis (LDA), Logit, Probit, proportional hazard and other similar statistical models do not account for the time-series behavior of financial variables, therefore they cannot distinguish between transitory and non-transitory changes in a firm's financial variables. In addition, the first three models assess the financial condition of a firm using data from a single period. As such, the models cannot account for the fact that a firm may be under performing for a series of years before it exhibits definite signs of financial distress.

Popular financial variables included in past financial distress models are generally non-stationary or exhibit strong positive correlation over time. It is well known that the unconditional variance of non-stationary variables increases over time, e.g., Dickey and Fuller (1979). Consequently, financial distress models based on such variables are expected to be non-stationary. Theodossiou and Kahya (1996) show that the ratios of net working capital to total assets, retained earnings to total assets, sales to total assets, operating income to total assets and market value of equity to book value of debt, used in Altman's (1968) popular Z-score model, as well as the ratios of total liabilities to total assets, receivables to sales, gross profit to sales and various proxies for size produce financial distress models with deteriorating forecasting performance over time.

The ability to predict business failures can be strengthened using models that account (1) for serial correlation in the data, (2) incorporate information from more than one period and (3) include stationary explanatory variables. The purpose of this paper is to develop and implement such a model. The model is based on the statistical methodology of time-series Cumulative Sums (CUSUM) developed by Theodossiou (1993) for predicting shifts in the mean of a multivariate time-series process. This paper extends significantly the work of Theodossiou (1993) by avoiding problems associated with non-stationary financial variables and the collection of the samples of failed and healthy firms. Moreover, the paper incorporates several refinements of the CUSUM model and focuses on the financial aspects rather than the statistical aspects of the model.

Financial distress models based on the CUSUM methodology assess sequentially the financial condition of a firm incorporating current and past information about the firm. A desirable characteristic of the CUSUM model is that it has very ''short memory'' in the case of past good performance, but ''long memory'' in the case of bad performance by the firm. A current bad performance will be erased from the memory of the model when the firm exhibits good performance for a series of years after the bad performance. The latter property makes the CUSUM model a more realistic tool for predicting business failures, because it imitates the practice used by bank loan officers and corporate raters.

The paper proceeds as follow. Section 2 elaborates on the statistical methodology of the CUSUM model. Section 3 provides a discussion on the sampling and the variables considered. Section 4 presents the estimation of the CUSUM model and several robustness tests for the model. Section 5 provides four applications of the model. The last section presents the summary and concluding remarks.

2. The CUSUM methodology

Let $X_{i,t} = [X_{i,1,t}, X_{i,2,t}, \ldots, X_{i,p,t}]$ to be a row vector of p attribute variables for the ith firm at time t with predictive ability with respect to financial distress. The sequence of attribute vectors $X_{i,1}, X_{i,2}, \ldots, X_{i,t}, \ldots$ for a healthy firm follows a "good" performance distribution with constant population mean over time.[3] For a financially distress (failing) firm, the sequence of its attribute vectors shifts (switches) gradually at some random time from the "good" performance distribution to a "bad" performance distribution. These shifts are initially small in magnitude and become larger as the firm approaches the point of economic collapse. A CUSUM model determines in an optimal manner the starting point of the shift and provides a signal of the firm's deteriorating condition as soon as possible after the shift.

2.1. *Time-series behavior of attribute vectors for firms*

The time-series behavior of the attribute variables for each firm, healthy or failed, can be adequately described by a finite order Vector Autoregressive model, VAR(k), as follows:

$$X_{i,t} = A_h + A_{f,s} + X_{i,t-1}B_1 + \cdots + X_{i,t-k}B_k + \varepsilon_{i,t}, \quad \text{for } s = 1, 2, \ldots m, \tag{1a}$$

$$A_{f,s} = 0, \quad \text{for healthy firms} \quad \text{and} \quad s > m, \tag{1b}$$

$$E(\varepsilon_{i,t}) = 0, \quad E(\varepsilon'_{i,t}\varepsilon_{i,t}) = \Sigma, \quad \text{and} \quad E(\varepsilon'_{j,r}\varepsilon_{i,t}) = 0, \tag{1c}$$

for $i \neq j$ and/or $r \neq t$,

where $\varepsilon_{i,t} = [\varepsilon_{i,1,t}, \varepsilon_{1,2,t}, \ldots, \varepsilon_{i,p,t}]$ is an independently distributed error vector with mean zero and variance-covariance matrix equal to $\Sigma, A_h = [A_{1,h}, A_{2,h}, \ldots, A_{p,h}]$ is a vector of intercepts for healthy firms, $A_{f,s} = [A_{1,f,s}, A_{2,f,s}, \ldots, A_{p,f,s}]$ are deviations from A_h associated with attribute vectors for failed firms extracted "s" years prior to failure and $B_l, \ldots, B_k$ are $p \times p$ matrices of VAR coefficients. The term $A_{f,s}$ captures permanent shifts in the mean structure of the variables due to financial distress. By construction, $A_{f,s}$ is equal to zero for all attribute vectors (observations) of the healthy firms. Also, $A_{f,s}$ is zero for observations of failed firms extracted prior to the starting point of the switch in the distribution of $X_{i,t}$ from the healthy population to the failed population (i.e., for $s > m$). Equation $E(\varepsilon'_{j,r}\varepsilon_{i,t},) = 0$, for $i \neq j$ and/or $r \neq t$, implies that the error term is uncorrelated across firms and time. For practical purposes the covariance matrix of the error term is specified to be equal in both groups, e.g., Marks and Dunn (1974) and Altman et al. (1977).

A necessary condition for the above VAR process to be stationary is that the roots of the polynomial $\det(I - B_1z - - B_kz^k) = 0$ lie outside the complex unit circle, where I is an identity matrix, z are the roots of the polynomial and det denotes the determinant, e.g., Judge et al. (1985), pp. 656–659. Stationarity implies that the variables are mean-reverting in the sense that when they depart from their mean values they return back to them in the near future. Stationarity of the attribute vectors $X_{i,t}$ also has significant implication regarding the robustness of financial distress models over time, e.g., Theodossiou and Kahya (1996).

Under stationarity of the VAR process, the unconditional mean of $X_{i,t}$ for healthy firms is equal to $\mu_h = A_h + \mu_h B_1 + \cdots + \mu_h B_k$, or $\mu_h = A_h(I - B_1 - \cdots - B_k)^{-1}$. Substitution of the first formula into equation (1a) gives

$$(X_{i,t} - \mu_h) = A_{f,s} + (X_{i,t-1} - \mu_h)B_1 + \cdots + (X_{i,t-k} - \mu_h)B_k + \varepsilon_{i,t}, \quad \text{for} \quad s = 1, 2, \ldots, m, \tag{2}$$

where $(X_{i,t} - \mu_h)$ denotes the deviations of a firm's attribute variables from their mean values in the healthy population. These deviations are functions of their past values (serial correlation), the financial distress term $A_{f,s}$, and the white noise error term, $\varepsilon_{i,t}$. The above formulation provides a more intuitive framework for interpreting the time-series behavior of financial variables, see also Theodossiou (1993).

2.2. *Time-series CUSUM model*

Based on the sequential probability ratio tests and the theory of optimal stopping rules, Theodossiou (1993) shows that the CUSUM model will provide a signal of the firm's deteriorating condition as soon as:

$$C_{i,t} = \min(C_{i,t-1} + Z_{i,t} - K, 0) < -L, \quad \text{for } K, L > 0, \tag{3}$$

where $C_{i,t}$ and $Z_{i,t}$ are respectively a cumulative (dynamic) and an annual (static) time-series performance score for the ith firm at time t, and K and L are sensitivity parameters that take positive values.[4]

The score $Z_{i,t}$ is a complex function of the attribute variables $X_{i,t}$ accounting for serial correlation in the data. It is calculated using the formula:

$$\begin{aligned} Z_{i,t} &= \beta_0 + (X_{i,t} - A_h - X_{i,t-1}B_1 - \cdots - X_{i,t-k}B_k)\beta_1 \\ &= \beta_0 + (A_{f,s} + \varepsilon_{i,t})\beta_1, \end{aligned} \tag{4}$$

$$\beta_0 = \left(\frac{1}{2}\right) A_f \Sigma^{-1} A_f{}' = D/2, \tag{5}$$

$$\beta_1 = (-1/D)\Sigma^{-1} A_f{}', \quad \text{and} \tag{6}$$

$$D^2 = A_f \Sigma^{-1} A_f{}', \tag{7}$$

where β_0 and β_1 are the CUSUM parameters and D is the Mahalanopis generalized distance of the error terms (i.e., the unpredictable component of the variables) in the healthy and failed samples. Note that for simplicity of notation $A_f \equiv A_{f,1}$. As shown in Appendix I, the annual performance score $Z_{i,t}$ has a positive mean of $D/2$ in the healthy population and a negative mean of $-D/2$ in the failed population, for $s = 1$. Moreover, the $Z_{i,t}$ scores are serially uncorrelated over time and have a variance of one in both the healthy and failed populations.

According to the CUSUM model, the overall performance of a firm at a given point in time is assessed by the cumulative score $C_{i,t}$. For as long as the firm's $Z_{i,t}$ scores are positive and greater than K, or $Z_{i,t} - K > 0$, the CUSUM score $C_{i,t}$ is set to zero indicating no change in the firm's financial condition. When the $Z_{i,t}$ scores fall below K, or $Z_{i,t} - K < 0$, the CUSUM score $C_{i,t}$ accumulates negatively. This accumulation continues for as long as $Z_{i,t} - K < 0$. A signal of the firm's changed condition is given at the time the CUSUM score $C_{i,t}$ falls below $-L$. Note that the CUSUM scores would increase and go back to zero if and only if the firm displayed $Z_{i,t}$ scores greater than K.[5]

2.3. *Sensitivity parameters for the CUSUM model*

The sensitivity parameters K and L determine the time between the occurrence and the detection of a change in the financial condition of a firm. The larger the value of K, the lower the probability of misclassifying a failing firm as healthy and the larger the probability of misclassifying a healthy firm as failed. The opposite is true with the parameter L.

Define:

$$P_f \equiv P(C_{i,t} > -L| \text{ failed and } s = 1), \quad \text{and} \tag{8a}$$

$$P_h \equiv P(C_{i,t} \leq -L| \text{ healthy}) \tag{8b}$$

to be respectively the percentages of failed and healthy firms in the population misclassified by the CUSUM model. These are also known as Type I and Type II errors and they are functions of the parameters K and L. The optimal values of K and L are derived by solving the following dynamic optimization problem:

$$\min_{K,L} EC = w_f P_f(K,L) + (1 - w_f)P_h(K,L), \tag{9}$$

where w_f and $w_h = (1 - w_f)$ are investors' specific weights attached to the error rates P_f and P_h. The EC is specified as a function of P_f, because the CUSUM model is developed for the purpose of predicting a shift in the mean of a firm's attribute vector from μ_h to $\mu_f \equiv \mu_{f,l}$, but not necessarily to any intermediate state.

The weights $w_f = \pi_f c_f/(\pi_f c_f + \pi_h c_h)$ and $w_h = \pi_h c_h/(\pi_f c_f + \pi_h c_h)$ are functions of the *a-priori* probabilities π_f and $\pi_h = 1 - \pi_f$, which measure the actual proportion of failed and healthy firms in the population, and the costs c_f and c_h associated with the misclassification of failed and healthy firms. Note that the *a-priori* probability for the financially distressed population π_f is smaller than that of the healthy population π_h. However, the cost of misclassifying a financially distressed firm (c_f) is larger than the cost of misclassifying a healthy firm (c_h). In the absence of specific weighing, the choice of equal weights ($w_f = w_h = 1/2$) appears to be a reasonable alternative. The EC criterion with equal weights is used within a neural network framework to select the profile of variables with the best overall forecasting performance. The error rates for various combinations of the parameters K and L used in the optimization of the above function are calculated using the jack-knife method.

3. Sampling and financial variables

3.1. Sampling methodology

The selection of the sample of financially distressed (failed) firms is based on debt default criteria, such as debt default or attempts to renegotiate debt with creditors and financial institutions. Information on debt default and debt renegotiation is gathered from various annual issues of the *Wall Street Journal Index* (*WSJI*). The time of failure is chosen as the first time the firm experienced one of the signs of failure.[6] The above definition of financial distress avoids many of the problems associated with the legal definition of business failure. The examples below provide the rationale for using debt default criteria to select financially distressed firms.

The 1978 federal Bankruptcy Code made it easy for firms to file petitions for Chapter 7 liquidation or Chapter 11 reorganization. As a result, many firms filed for bankruptcy liquidation or reorganization for reasons other than financial distress. For example, in 1982, the Manville Corp. filed under Chapter 11 as a way of dealing with lawsuits from individuals claiming exposure to its asbestos products. In 1987, Texaco filed under Chapter 11 to reduce its liability to Pennzoil. In 1994, Petrie Stores Corp. received a favorable ruling from the IRS allowing a tax-free liquidation. None of these companies exhibited any signs of financial distress prior to filing for bankruptcy. On the other hand, many financially distressed firms never file for bankruptcy because of acquisition. For example, in 1980, American Motors Corp. (AMC) was rescued by Renault while experiencing serious debt servicing problems. In 1987, AMC was acquired by the Chrysler Corp. Similarly, in 1986, Clevepak Corp. was acquired by the Madison Management Group, Inc., five months after suspending payment of principal on debt.

These examples show that the legal definition of failure results in ''contaminated'' healthy and failed samples. That is, the failed sample will include firms that filed for bankruptcy for reasons other than financial distress and will disregard financially distressed firms that never filed for bankruptcy. The latter firms may be included in the healthy sample. Moreover, many financially distressed firms file for bankruptcy and operate under a reorganization plan for several years before filing for bankruptcy liquidation. This makes the determination of the timing of failure and collection of data a problematic one. The use of contaminated samples and incorrect information on the timing of failure will distort the distributional properties of the financial variables in the healthy and failed samples and is likely to impair the forecasting ability of the models.

The samples obtained using the debt default criteria includes 117 healthy firms and 72 failed firms. Data for the firms are extracted from the 1993 annual industrial and research COMPUSTAT tapes and span the period 1974–1991. The sample of healthy firms is compiled from a sample of 150 firms collected randomly from the population of about

1,000 manufacturing and retailing firm listed on the NYSE and the AMEX in 1992. Note that this sample is large enough to provide a good coverage of the population. Twenty-two of the firms are dropped from the sample because of non-continuous data and/or a few annual observations. The remaining 128 firms are thoroughly screened for signs of financial distress using the annual volumes of the *WSJI* for the period 1978–1995. Eleven of these firms are found to exhibit signs of financial distress; thus, they are classified as failed. The remaining failed firms are identified using debt default criteria from a population of about 300 manufacturing and retailing firms delisted from the NYSE and AMEX during the period 1982–1992 because of bankruptcy liquidation, bankruptcy reorganization, privatization, merger, and acquisition. OTC firms are not considered because they are generally smaller than NYSE and AMEX firms and, as such, their financial attributes with respect to bankruptcy are expected to be different, e.g., Edmister (1972). Moreover, petroleum (SIC = 2911) and mining firms (SIC = 3312, 3330 and 3334) are not considered because they possess financial attributes that are statistically different from those of other manufacturing firms.

3.2. Financial variables

The variables considered are mostly derived from the broad class of financial ratios found to be significant explanatory variables in past financial distress models. Appendix II provides a list of the variables, the formulas used to compute their values and citations for a sample of studies that considered the variables. The variables are classified into the categories of liquidity, profitability, financial leverage, size, and other variables. In addition to the levels, the paper considers first differences (changes) in the variables over time. First differences provide useful information regarding financial distress. Moreover, they are preferable to variables' levels because levels are generally non-stationary over time.

4. Model identification and estimation

4.1. Model identification

The identification of the best CUSUM model is accomplished using a neural network type search procedure based on the expected cost (EC) function, i.e., by minimizing the EC function given by equation (9). Its explanatory variables are chosen from a set of 54 variables, which includes the 27 variables listed in Appendix II and their first differences. Another criterion used by the search procedure is stationarity of the models over time. Interestingly, popular financial variables included in past financial distress models produce non-stationary models with deteriorating performance over time, e.g., Theodossiou and Kahya (1996). Clearly, such models are not acceptable.

The set of 54 variables considered could generate an extremely large number of profiles of financial variables.[7] Searching all possible profiles is not desirable. For practical purposes, the search procedure is programmed to allow for one explanatory variable from each major category of variables to enter a model at a time. The latter approach is reasonable, because the inclusion of two or more variables from the same category is not expected to improve significantly a model's performance.

The best stationary CUSUM model produced by the search procedure includes four explanatory variables. These are the change in the logarithm of deflated total assets, the change in the ratio of inventory to sales, the change in the ratio of fixed assets to total assets

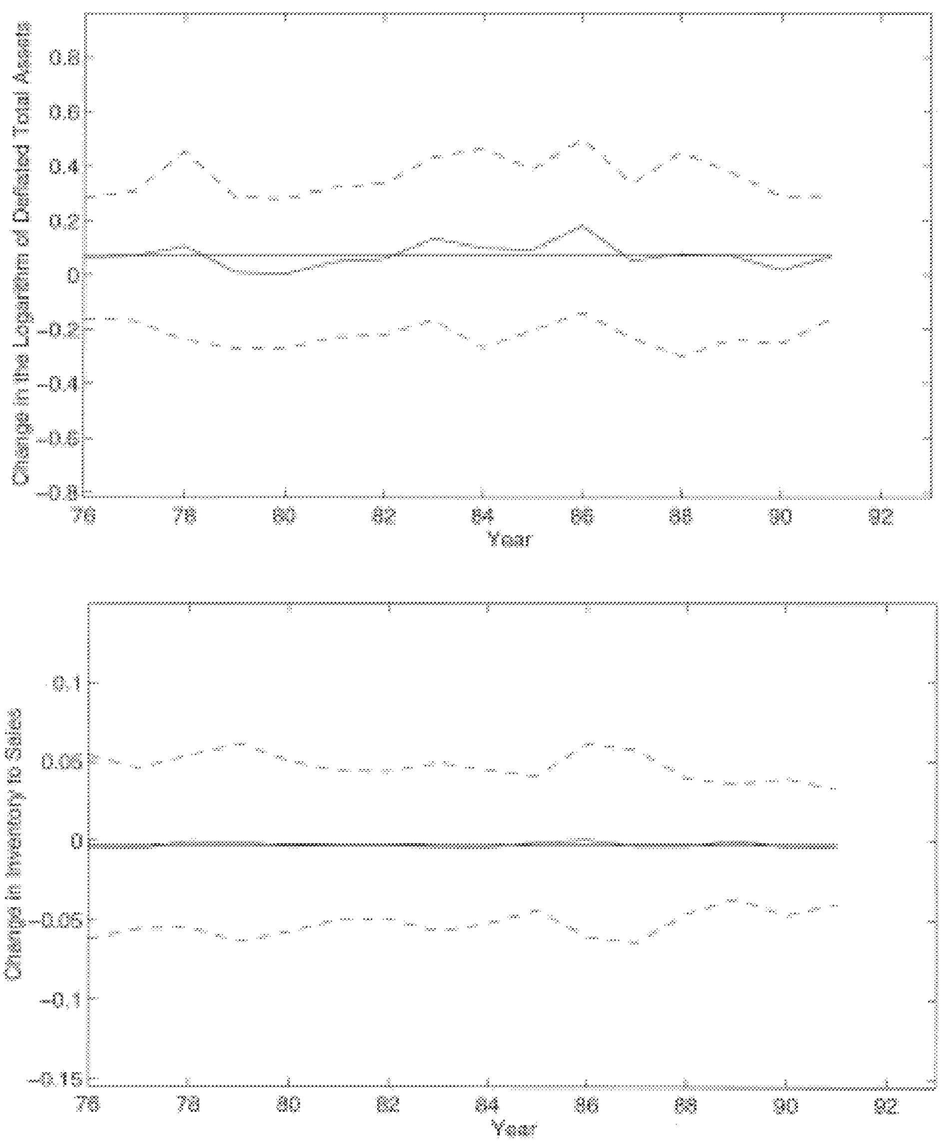
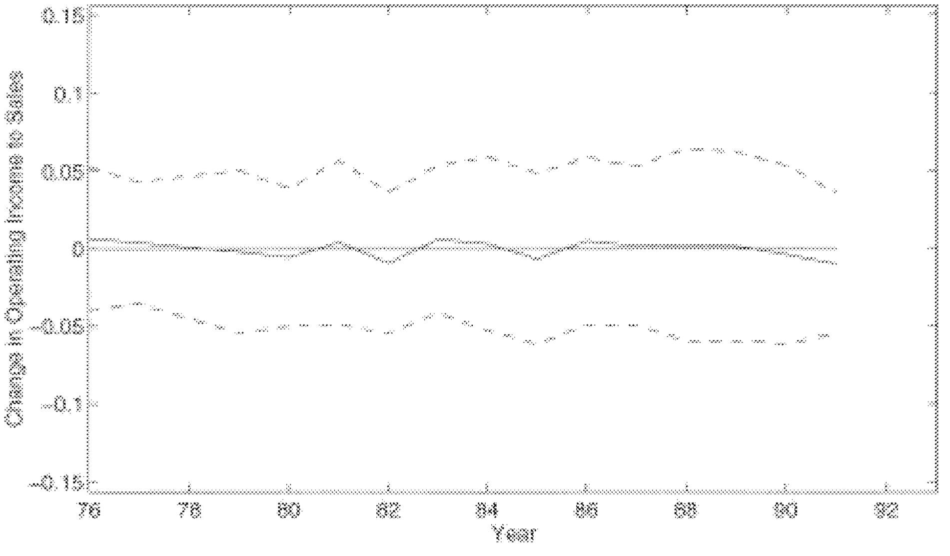


Figure 1. Means and standard deviations of the variables in the healthy sample.

and the change in the ratio of operating income to sales. Interestingly, the above model exhibits at least as good average performance over time as the best non-stationary model.[8]

Figure 1 illustrates the annual sample means and standard deviations of the four variables for the sample of 117 healthy firms. The standard deviations for the variables are presented in the form of plus and minus two standard deviations from the means. As such, they provide a distributional range for 95% of their values. The straight line gives the



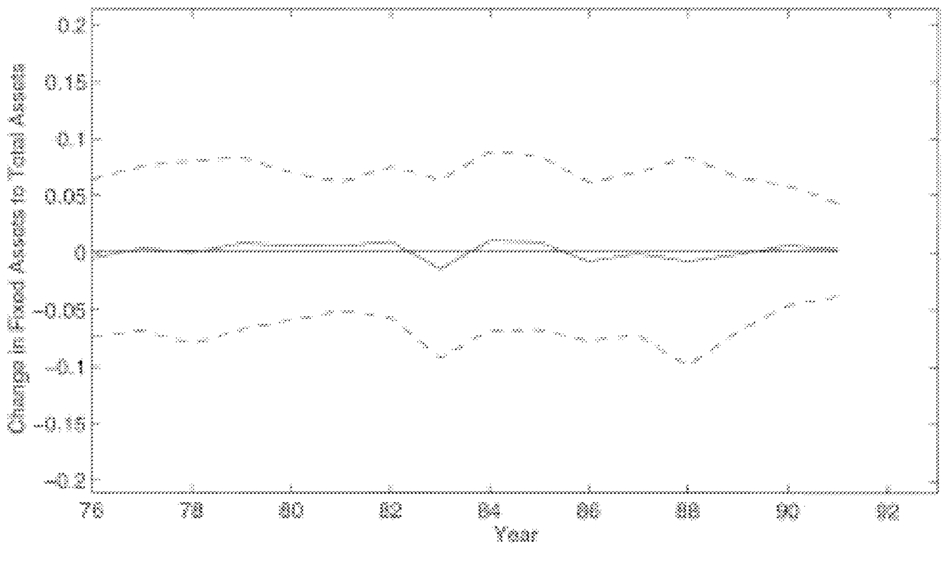


Figure 1. (*Continued*)

overall mean of the variables using the data for the entire period 1976–1991. The results indicate that all four variables are relatively stable over time.

Figure 2 illustrates the means and standard deviations of the four variables by year prior to failure for the sample of 72 failed firms. The straight line gives the overall mean of the variables for the 117 healthy firms. The means of the variables in the failed sample are lower for the change in the logarithm of deflated total assets, the change in the ratio of

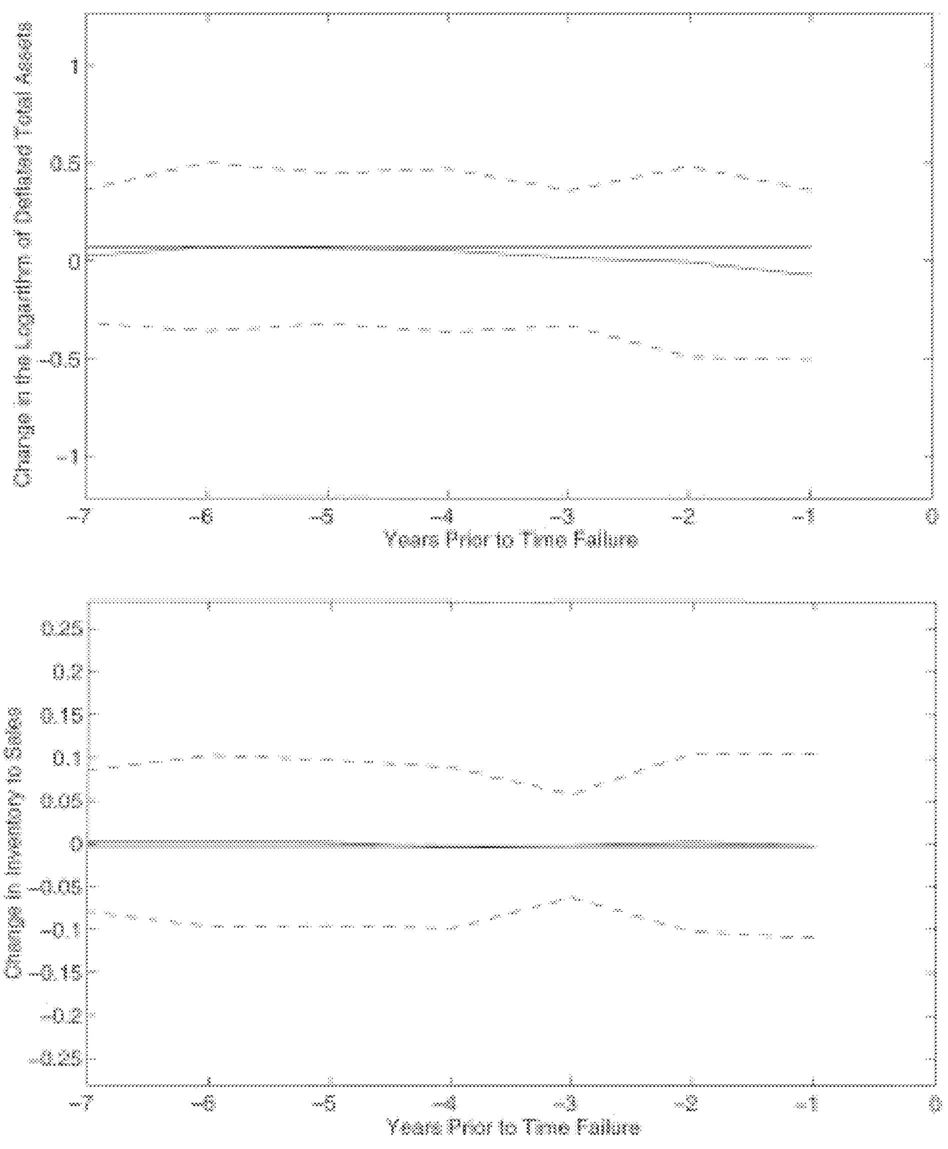


Figure 2. Means and standard deviations of the variables in the failed sample.

fixed assets to total assets, and the change in the ratio of operating income to sales, and higher for the change in the ratio of inventory to sales. These means, at one year prior to failure (i.e., $s = 1$), are statistically different from their respective overall means in the healthy sample, except for the mean of the change in the ratio of fixed assets to total assets. The latter variable, however, in combination with the other three variables improves the predictive ability of the model.

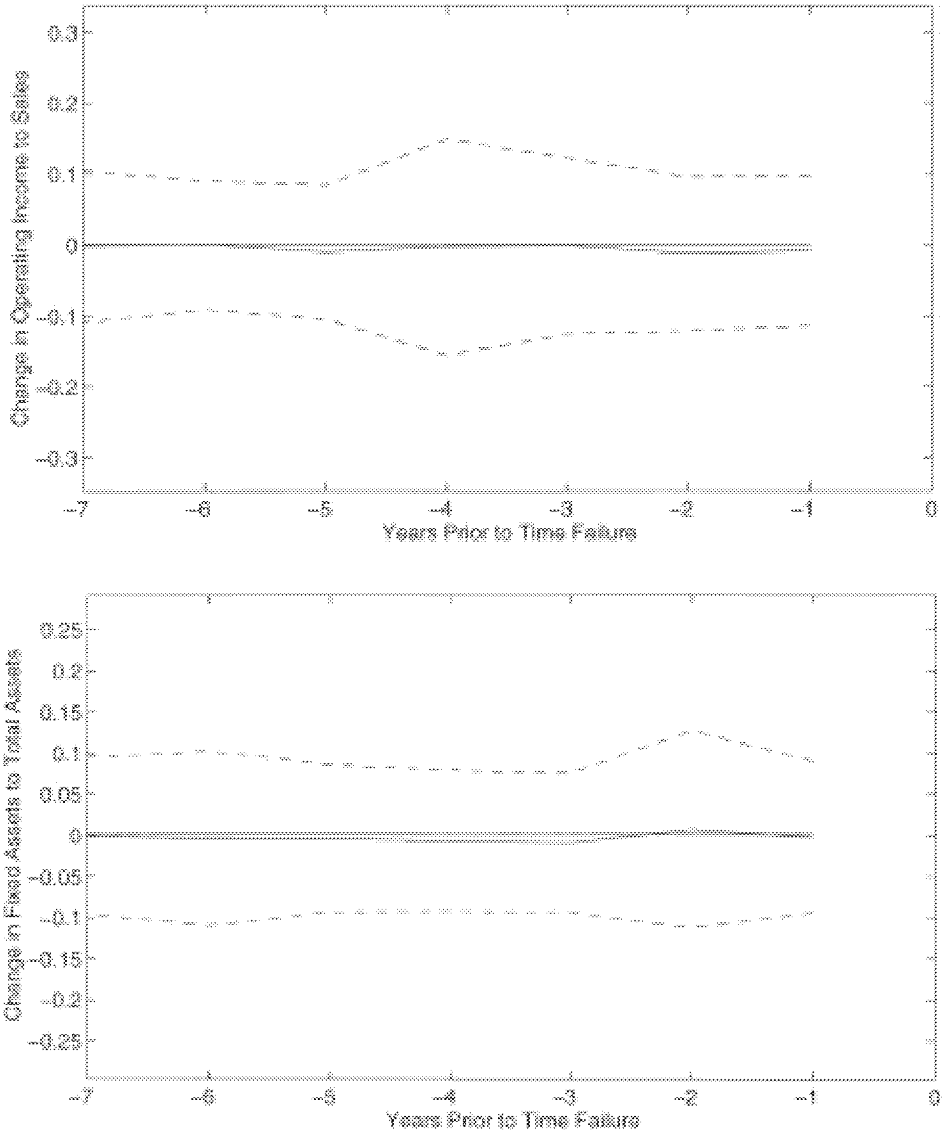


Figure 2. (*Continued*)

4.2. *Time-series behavior of explanatory variables*

The VAR estimates for the four explanatory variables are obtained by fitting equations (1a) to the data for the 72 failed firms and 117 healthy firms over the period 1974–1991. Pooling of the data in the estimation is necessary because of the small number of yearly observations for each firm and for homogeneity reasons. In the best case, 18 yearly observations are available while on many occasions firms had a few yearly observations. The VAR estimates are obtained by maximizing the log-likelihood function of the pooled sample, e.g., Johansen (1995), p. 18. Due to random sampling, the log-likelihood function is specified as the sum of individual firm log-likelihood functions.

The identification of the order of the VAR model is performed using the Akaike's information criterion; i.e., by minimizing $\mathrm{AIC} = \log(\det(\Sigma)) + (2M/NT)$, where M represents the number of estimated VAR coefficients, and NT represents the number of yearly observations in the pooled sample, and Σ is the estimate of the error covariance matrix based on the residuals of the pooled sample, denoted by $e_{i,t}$. That is, $\Sigma = (\Sigma e'_{i,t}e_{i,t})/(NT-5)$. The analysis of the data by means of AIC yielded a first order VAR model, i.e., VAR(1). It is important to note that the estimation and identification of the order of the VAR model are performed automatically by the neural network procedure described previously.

The estimated VAR(1) model is as follows:

$$X_{i,t} = A_h + A_f + X_{i,t-1}B_1 + e_{i,t},$$

where

$$\begin{array}{lcccc} & \Delta \text{ in log of Total Assets} & \Delta \text{ in Inventory to Sales} & \Delta \text{ in Fixed Assets to Total Assets} & \Delta \text{ in Operating Income to Sales} \end{array}$$

$$A_h = 10^{-2}\begin{bmatrix} 5.2824 & -0.3218 & -0.0088 & 0.0717 \\ (13.7) & (-4.69) & (-0.09)^* & (0.84)^* \end{bmatrix}$$

$$A_f = 10^{-2}\begin{bmatrix} -12.0413 & 0.1391 & -0.2003 & -1.0674 \\ (-6.26) & (0.38)^* & (-0.42)^* & (-2.52) \end{bmatrix}$$

$$B_1 = \begin{bmatrix} 0.2863 & 0.0000 & 0.0171 & -0.0166 \\ (14.87) & (0.00) & (3.79) & (-3.92) \\ -0.3727 & -0.2104 & 0.0000 & -0.0817 \\ (-3.59) & (-11.18) & (0.00) & (-3.58) \\ 0.1783 & 0.0000 & 0.0000 & 0.0000 \\ (4.62) & (0.00) & (0.00) & (0.00) \\ 0.3417 & 0.0000 & 0.0000 & -0.1907 \\ (3.91) & (0.00) & (0.00) & (-10.0) \end{bmatrix}$$

and Δ denotes the first difference or one-lag operator.

The VAR coefficients B_1 (parentheses include the t-values of the estimates) provide information on how the variables relate to their past values as well as to past values of the other variables. Statistically insignificant autoregressive coefficients are set equal to zero. In this respect, each equation is re-estimated using only past values for the variables that exert a statistically significant relationship on current values of each variable. Estimates of $A_{f,s}$, for $s = 2, \ldots, m$, are available upon request.

The pooled variance-covariance matrix in the healthy and failed samples (at one year prior to failure) is estimated from the residuals using the formula:

$$\Sigma_p = ((N_h - 5)\,\Sigma_h + (N_f - 5)\,\Sigma_f)/(N_h + N_f - 10), \tag{10}$$

where $N_h = 1,958$ is the total number of yearly observations for the 117 healthy firms, $N_f = 71$ is the number of observations extracted at one year prior to failure, $\Sigma_h = (\Sigma e'_{i,t} e_{i,t})/(N_h - 1)$ is 4×4 variance-covariance matrix of $e_{i,t}$ in the healthy sample and $\Sigma_f = (\Sigma e'_{i,t} e_{i,t})/(N_f - 1)$ is 4×4 variance-covariance matrix of $e_{i,t}$ in the failed sample using the residuals at one year prior to failure.

$$\Sigma_p = 10^{-2} \begin{bmatrix} 2.1916 & 0.1146 & -0.0978 & 0.0460 \\ 0.1146 & 0.0666 & 0.0021 & -0.0143 \\ -0.0978 & 0.0021 & 0.1319 & -0.0148 \\ 0.0460 & -0.0143 & -0.0148 & 0.0733 \end{bmatrix}.$$

The pooled variance-covariance is the proper measure to use into equations (4)–(7), because the CUSUM model is developed for the purpose of predicting a shift in the mean of a firm's attribute vector from μ_h to μ_f, but not to any intermediate state, e.g., Amemiya (1981), p.1509.

Substitution of A_h, A_f, B_1, Σ_p, into equations (4)–(7) yields:

$D = 0.9387$

$\beta_0 = 0.2813$

	Δ in log of Total Assets	Δ in inventory to sales	Δ in Fixed Assets to Total Assets	Δ in Operating Income to Sales
$\beta_1 =$	[6.5815	-11.4976	10.7195	7.8873].

The estimated parameters for β_0, β_1, A_h, and B_1 along with equation (4) can be used to calculate a firm $Z_{i,t}$ scores as follows:

$$Z_{i,t} = \beta_0 + \beta_1(X_{i,t} - A_h - X_{i,t-1}B_1).$$

The coefficients β_1 measure the impact of the variables on the firm's performance score $Z_{i,t}$. Moreover, these coefficients provide an economic understanding of the relationship between the variables and the financial performance of the firm.

Specifically, the coefficient associated with the variable of the change in the logarithm of deflated total assets has a positive sign implying a positive relationship between the variable and the financial performance of the firm. That is, higher growth rates for a firm are indicative of good financial performance and vice versa.

The coefficient associated with the change in the ratio of inventory to sales has a negative sign implying a negative relationship between the variable and the performance of the firm. The ratio of inventory to sales measures the management's ability to turn inventory into sales, e.g., Theodossiou (1993) and Theodossiou et al. (1996). Higher values for this ratio are indicative of management inefficiency. Therefore, increases in this ratio will affect the financial performance of the firm negatively.

The coefficient associated with the change in fixed assets to total assets has a positive sign. Fixed assets (property, plant and equipment) are mainly used by firms to produce and distribute goods and services. Financially distressed firms frequently sell fixed assets to improve their liquidity position. On the other hand, healthy firms increase their fixed asset position by expanding or modernizing their plants. Therefore, decreases in this ratio are likely to be associated with deteriorating financial performance for the firm and vice versa.

Finally, the coefficient associated with the change in the ratio of operating income to sales has positive signs implying a positive relationship between profitability and the financial performance of the firm.

Figure 3 illustrates the time path of the mean of $Z_{i,t}$ scores in the failed sample starting from six years prior to failure down to one year prior to failure. The horizontal lines at $D/2$ and $-D/2$ denote the means of $Z_{i,t}$ in the healthy and failed (for $s = 1$) samples, respectively. Note that the average scores for failed firms at six years prior to failure are

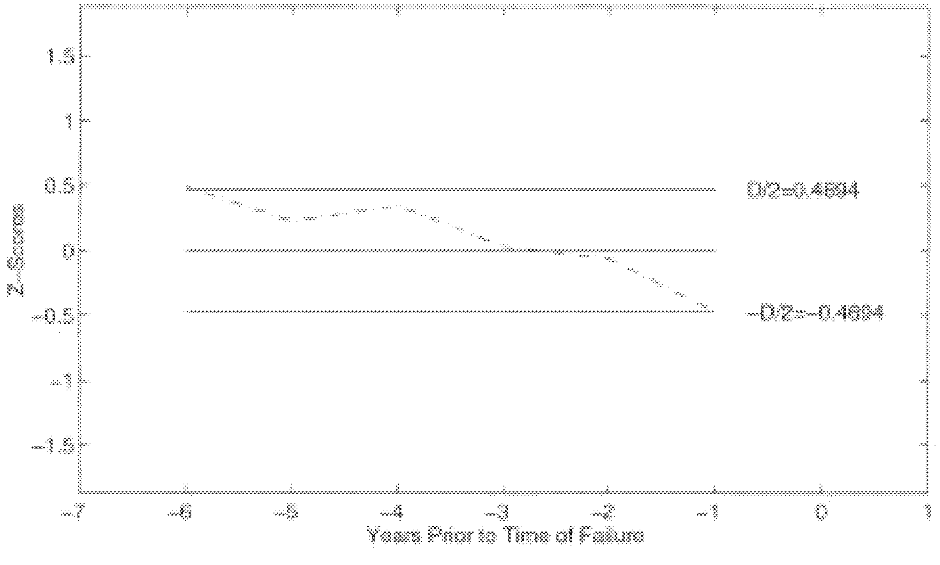


Figure 3. Means of the Z-Scores for failed firms.

close to the mean in the healthy sample. As the financial condition of the firms deteriorates, they move toward the failed sample mean of $-D/2$.

The CUSUM scores $C_{i,t}$ for each firm are calculated recursively using the formula

$$C_{i,t} = \min(C_{i,t-1} + Z_{i,t} - 0.0587, 0) < -0.8214.$$

A firm will be classified as failing once its CUSUM score falls below $-L = -0.8214$. Details on the derivation of the optimal values of $K = 0.0587$ and $L = 0.8214$ are presented below.

4.3. *Optimal values for K and L*

The EC criterion is used to determine the optimal sensitivity parameters of the CUSUM and evaluate the forecasting performance of the model in the failed and healthy samples. As a first step in applying the EC criterion, the error rates of each model P_f and P_h are computed for different combinations of values for K and L via the jack-knife method with 250 replications.

During each replication one healthy and one failed firm are randomly dropped from the data and all CUSUM parameters are re-estimated. Equations (3)–(7) are then used to calculate the CUSUM scores over time for the held-back firms. Next, all yearly observations for the held-back healthy firm and the last observation ($s = 1$) for the held-back failed firm are reclassified using their respective CUSUM scores. A tally of the number of misclassified observations is kept. P_h is computed by dividing the number of misclassified observations by the total number of observations of all 250 held-back healthy firms. P_f is computed by dividing the number of misclassified failed observations by 250.

The jack-knife method avoids the problem of bias in the error rates resulting from the model being tested on the same data from which it has been derived.[9] The jack-knife method is superior to the holdout method, because it permits the use of all available data in the estimation, e.g., McLachlan (1992), pp. 341–342.[10]

Equation (9) is then used to compute the model's expected cost function EC for values of the weight w_f ranging between 0.40 and 0.60 with increments of 0.05 for all combinations of K and L. For each value of w_f, the K and L combination is chosen so that EC is minimized. The optimal values of K and L and their respective error rates for a selected range of w_f are presented in Table 1. Note that the optimal values for $w_f = w_h = 1/2$ are $K = 0.0587$ and $L = 0.8214$. The last three columns of the panel present the error rates in the failed group using the CUSUM scores corresponding to two, three and four years prior to failure.

For comparison purposes, jack-knife estimates for the error rates are also computed for the LDA and Logit models using the same set of explanatory variables. The results for the error rates of LDA and Logit are presented in panels B and C, respectively. Estimates for the LDA and Logit models and other statistics are available upon request.

Table 1. Error rates for the CUSUM, LDA and logit models

Panel A—Optimal Values of K and L and Error Rates for the CUSUM Model

w_f	K	L	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
0.40	0.0939	1.7601	0.0684	0.3099	0.5556	0.6667	0.7183
0.45	0.1056	1.2907	0.1170	0.2394	0.4861	0.5417	0.6338
0.50	0.0587	0.8214	0.1706	0.1831	0.4028	0.4583	0.6056
0.55	0.0117	0.5867	0.2028	0.1549	0.3750	0.3750	0.5352
0.60	0.0821	0.7040	0.2201	0.1408	0.3750	0.3750	0.5352

Panel B—Optimal Cut-off Points (L) and Error Rates for the LDA Model

w_f	L	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
0.40	−0.2288	0.1619	0.3239	0.4722	0.5556	0.7183
0.45	−0.2288	0.1619	0.3239	0.4722	0.5556	0.7183
0.50	−0.2053	0.1736	0.3099	0.4722	0.5556	0.7183
0.55	−0.0997	0.2191	0.2676	0.4583	0.5139	0.6338
0.60	−0.0997	0.2191	0.2676	0.4583	0.5139	0.6338

Panel C—Optimal Cut-off Points (L) and Error Rates for the Logit Model

w_f	L	P_h	P_f	$P_{f,2}$	$P_{f,3}$	$P_{f,4}$
0.40	0.0537	0.1241	0.3662	0.5139	0.5972	0.7606
0.45	0.0478	0.1583	0.3239	0.5139	0.5694	0.7042
0.50	0.0478	0.1583	0.3239	0.5139	0.5694	0.7042
0.55	0.0362	0.2666	0.2254	0.4167	0.4444	0.5775
0.60	0.0362	0.2666	0.2254	0.4167	0.4444	0.5775

Panel D—Ratio of CUSUM to LDA Expected Cost

w_f	$s=1$	$s=2$	$s=3$	$s=4$
0.40	0.7278	0.9205	0.9636	0.8541
0.45	0.7328	0.9388	0.9087	0.8478
0.50	0.7315	0.8877	0.8625	0.8702
0.55	0.7179	0.8483	0.7803	0.8623
0.60	0.6952	0.8632	0.7906	0.8745

Panel E—Ratio of CUSUM to Logit Expected Cost

0.40	0.7468	0.9402	0.9821	0.8672
0.45	0.7390	0.8893	0.8973	0.8652
0.50	0.7334	0.8529	0.8642	0.8999
0.55	0.7234	0.8521	0.8164	0.8812
0.60	0.7135	0.8778	0.8386	0.9030

Notes: P_h is the percentage of healthy firms misclassified by the models. $P_{f,s}$ is the percentage of failed firms misclassified by the models using observations extracted "s" years prior to failure, for $s = 1, 2, \ldots, 4$. Optimal values of K and L are those values that minimize the expected cost function of each model for a given weight w_f.

Panels D and E of the table present the ratio of expected cost of CUSUM to those of LDA and Logit models, respectively. The results show that the CUSUM model outperforms both the LDA and Logit models. For example, if one were to consider the class of investors who put equal weight on the two types of errors, the cost associated with the use of the CUSUM model would be 73.15% that of the LDA model for $s = 1$, 88.77% for $s = 2$, 86.25% for $s = 3$, and 87.2% for $s = 4$.

4.4. Stationarity of the VAR model

A necessary condition for the estimated VAR process to be stationary is that the roots of the polynomial $\det(I - B_1 z) = 0$ lie outside the complex unit circle. The latter condition is met if the length of the smallest root is greater than one, i.e., $\min |z| > 1$. The lengths of the polynomial roots are 3.5192, 4.7280, 5.9546, and 87.3166. Interestingly, all roots are greater than one. This coupled with the fact that the means and variances of the variables are bounded (e.g., figures 1 and 2) provides strong support for the hypothesis that the VAR process as well as the CUSUM model are stationary.

Figure 4 presents graphical illustrations of the annual error rates (Type II error) of the CUSUM model in the sample of 117 healthy firms for the period 1978–1991.[11] The straight line provides the model's average error rate for the entire period, which is 17.29%. It appears that the CUSUM model exhibits no time trend. Thus, it is stationary (robust) over time. The following regression further assesses the stationarity of the model over time:

$$\underset{}{\mathrm{ERR}_t} = \underset{(0.59)^*}{0.2100} - \underset{(-0.05)^*}{0.0002t}, \qquad R^2 = 0.0002,$$

where ERR_t is the error rate for year t and $t = 78, \ldots, 91$. Note that the error rates are expressed in decimal form and parentheses include the t-values of the estimates. The slope of the regression, $\partial \mathrm{ERR}_t / \partial t$, gives the annual change (growth) in the error rates. In the presence of an upward time-trend, the slope of the regression is expected to be positive and statistically significant. The regression slope is statistically insignificant at the 5% level,

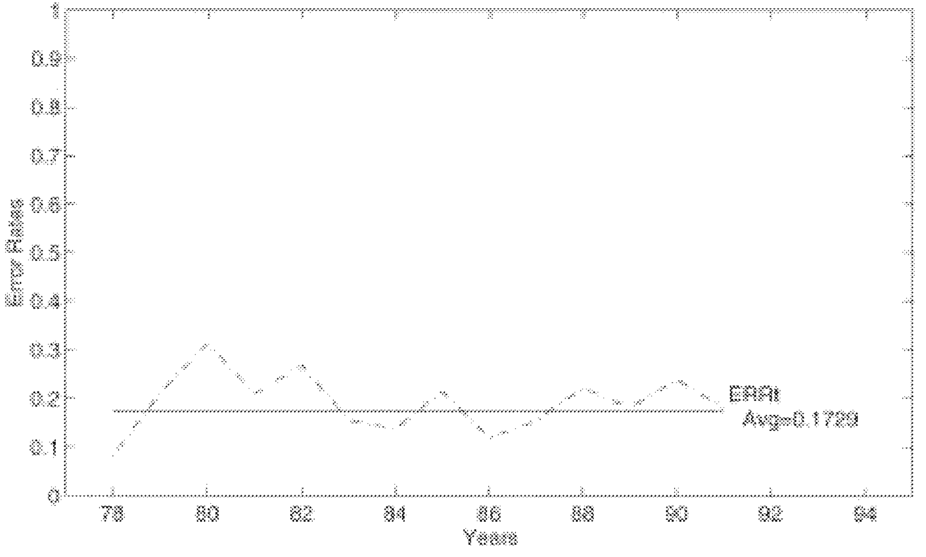


Figure 4. Annual error rates of the CUSUM model for healthy firms.

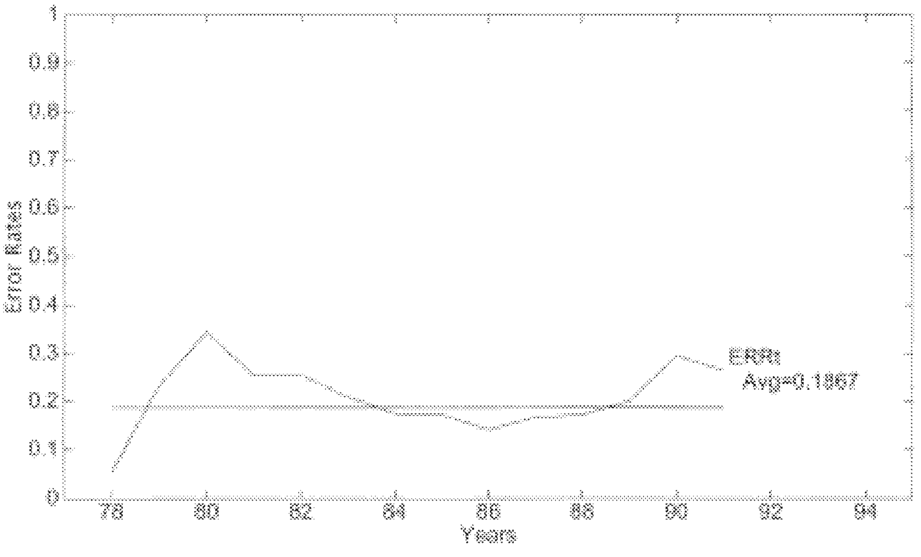


Figure 5. Annual error rates of the CUSUM model for the S&P400 sample

indicating no time trend. This finding is also supported by the low R-square value of the regression.

To further evaluate the performance of the CUSUM model over time, a new sample of 279 healthy manufacturing firms included in the S&P400 index is used. Note that this sample and the estimation sample include seven common firms only. Figure 5 provides an illustration of the model's error rates over time for the S&P400 sample. The average error rate for the CUSUM model is 18.67%. The regression equations

$$\mathrm{ERR}_t = \underset{(0.11)^*}{0.0444} + \underset{(0.40)^*}{0.0020t}, \qquad R^2 = 0.0129,$$

where ERR_t is the error rate for year t and $t = 78, \ldots, 91$, reaffirm the previous findings that there is no time trend in the error rates of the three models, thus they are robust over time.

6. Summary and conclusions

This paper develops a stationary financial distress model based on the statistical method of Cumulative Sums (CUSUM) for predicting shifts in the mean of multivariate time-series processes. The model distinguishes between changes in the financial variables of a firm that are the result of serial correlation and changes that are the result of permanent shifts in the mean structure of the variables due to financial distress.

The model's explanatory variables include the change in the logarithm of deflated total assets, the change in the ratio of inventory to sales, the change in the ratio of fixed assets to total assets and the change in the ratio of operating income to sales. The CUSUM coefficients associated with the variables of growth of deflated total assets, change in the ratio of fixed assets to total assets, and change in the ratio of operating income to sales have positive signs implying a positive relationship between the variables and the financial performance of the firm. The coefficient for the change in the ratio of inventory to sales has a negative sign implying a negative relationship. These results for the parameter estimates are consistent with what one would expect. Generally, financially distressed firms experience decreases in their growth, profitability, and fixed assets, and increases their inventory levels relative to healthy firms.

Interestingly, none of the popular financial variables included in past financial distress models enters the model as an explanatory variable. Many of these variables exhibit strong positive serial correlation and in many cases near unit root behavior. Consequently, the inclusion of such variables in the CUSUM model or models based on other statistical methods produces non-stationary financial distress models with deteriorating forecasting performance over time, e.g., Theodossiou and Kahya (1996). Nevertheless, none of these variables produces better average classification performance.

The CUSUM model can be viewed as the dynamic extension of Discriminant analysis, a statistical technique employed in many business failure prediction studies. A desirable feature of the CUSUM model is that it has a very short ''memory'' with respect to a firm's good performances over the years, but a long ''memory'' in case of bad performances. The model's memory feature makes it sensitive to negative changes in a firm's financial condition. Consequently, it promptly alerts the financial analyst who may then undertake a closer investigation and assessment of the firm.

A comparison of the performance of the CUSUM model to Discriminant analysis and Logit shows the CUSUM model to be clearly superior. The model presented in this paper can be used by business loan officers, investors, corporate raters and other to asses the financial performance of US manufacturing and retailing firms traded on the AMEX and the NYSE. Moreover, the methodology on which the model is based can be applied to areas such as the rating corporate or municipal bonds, the assessment of the financial performance of commercial banks and/or savings and loans associations, and the prediction of the debt service problems of debtor countries.

Appendix I

It follows from equations (4) and (5) that $Z_{i,t}$ is equal to:

$$\begin{aligned} Z_{i,t} &= \beta_0 + A_{f,s}\beta_1 + \varepsilon_{i,t}\beta_1 \\ &= D/2 + A_{f,s}\beta_1 + \varepsilon_{i,t}\beta_1, \quad \text{for } s = 1, 2, \ldots, m. \end{aligned}$$

For healthy firms $A_{f,s} = 0$ and

$$Z_{i,t} = D/2 + \varepsilon_{i,t}\beta_1.$$

For observations of failed firms at one year prior to failure (i.e., for $s = 1$)

$$D/2 + A_f\beta_1 = D/2 - D = -D/2$$

and

$$Z_{i,t} = -D/2 + \varepsilon_{i,t}\beta_1.$$

It easily follows from the above equations that the mean of $Z_{i,t}$ for the healthy firms is $D/2$ and for failed firms is $-D/2$. This is because the mean of $\varepsilon_{i,t}$ is zero.

Moreover, because the residuals are uncorrelated over time, individual $Z_{i,t}$ scores for healthy and failed firms are expected to deviate randomly over time around their population means. Further more, the variance of $Z_{i,t}$ in the healthy and failed populations are

$$\begin{aligned}\mathrm{var}(Z_{i,t}|h) &= \mathrm{var}(Z_{i,t}|f) = E\beta_1\varepsilon'_{i,t}\varepsilon_{i,t}\beta_1 \\ &= \beta'_1\Sigma\beta_1 = (1/D^2)A_f\Sigma^{-1}\Sigma\Sigma^{-1}A'_f = (1/D^2)D^2 = 1.\end{aligned}$$

Appendix II–financial variables considered

Variables	Computation	Proxy	Used in
Cash to current liabilities	V1/V5	Liquidity	Beaver (1966), Edmister (1972), Gombola et al. (1966)
Cash to total assets	V1/V6	Liquidity	Beaver (1966), Gombola et al. (1987)
Current assets to current liabilities	V4/V5	Liquidity	Beaver (1966), Altman et al. (1977) Gombola et al. (1966)
Current assets to total assets	V4/V6	Liquidity	Beaver (1966), Lo (1986), Gombola et al. (1987)
Net working capital to total assets	(V4-V5)/V6	Liquidity	Beaver (1966), Altman (1968), Ohlson (1980), Theodossiou (1993)
Net working capital to sales	(V4-V5)/V12	Liquidity	Edmister (1972)
Quick assets to current liabilities	(V4-V3)/V5	Liquidity	Beaver (1966)
Gross profit to sales	(V12-V41)/V12	Profitability	
Net income to book value of equity	V172/V216	Profitability	
Net income to fixed assets	V172/V8	Profitability	
Net income to total assets	V172/V6	Profitability	Beaver (1966), Ohlson (1980), Lo (1986), Gombola et al. (1987)

Appendix II (*Continued*)

Operating income to fixed assets	V13/V8	Profitability	
Operating income to sales	V13/V12	Profitability	Theodossiou et al. (1996)
Operating income to total assets	V13/V6	Profitability	Altman (1968), Altman et al. (1977)* Theodossiou (1993)
Retained earnings to total assets	V36/V6	Long-term Profitability	Altman (1968), Altman et al. (1977)
Long-term debt to total assets	V9/V6	Financial Leverage	Beaver (1966), Altman (1968)
Total Liabilities to total assets	V181/V6	Financial Leverage	Ohlson (1980), Gombola et al. (1987) Theodossiou et al. (1996)
MVE to total liabilities	(V24*V25)/V181	Market Structure	Altman (1968)
Logarithm of deflated fixed assets	log(100*(V8/PPI))	Size	
Logarithm of deflated sales	log(100*(V12/PPI))	Size	Pastena and Ruland (1986)
Logarithm of deflated total assets	log(100*(V6/PPI))	Size	Altman et al. (1977), Ohlson (1980), Lo (1986), Theodossiou et al. (1996)
Logarithm of number of employees	log(V29)	Size	
Accounts receivable to current assets	V2/V4	Management Efficiency	
Accounts receivable to sales	V2/V12	Management Efficiency	Beaver (1966), Gombola et al. (1987)*
Fixed assets to total assets	V8/V6	Operating Leverage	Theodossiou (1993)
Inventory to sales	V3/V12	Management Efficiency	Beaver (1966), Edmister (1972), Theodossiou (1993), Theodossiou et al. (1996)
Sales to total assets	V12/V6	Activity	Altman (1968), Gombola et al. (1987)

Notes: This paper also considers the annual changes in the values of the above variables from year $t-1$ to year t. The citations indicate studies that considered the variables. *Specifically, Altman (1968) and Altman et al. (1977) used the ratio of EBIT (Earnings Before Interest and Taxes) to total assets and Gombola et al. (1987) used the reciprocal of the ratio of accounts receivable to sales. The numbers following the letter ''V'' are the numbers assigned to the variables in the COMPUSTAT manual. PPI is the producer price index.

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Notes

1. A simple measure of serial correlation is given by autocorrelation function. This is calculated using the formula $\rho(s) = \mathrm{Cov}(X_{i,t}, X_{i,t-s})/\mathrm{Var}(X_{i,t})$, where $X_{i,t}$ is the value of the ith variable at time t, $\mathrm{Var}(X_{i,t})$ is the variance of $X_{i,t}$, and $\mathrm{Cov}(X_{i,t}, X_{i,t-s})$, for $s = 1, 2, \ldots$, is the covariance between current and past values (s-lags) of the variables. For stable (stationary) time-series processes $|\rho(s)| < 1$, where $| \, |$ denotes the absolute value operator. A multivariate extension of the autocorrelation function can be found in Lütkepohl (1993), pp. 25–26.
2. In general the larger the value of $\rho(s)$ the greater the persistence of deviations of the variables from their means over time and the longer the memory of the process. Note that for non-stationary or random walk time-series processes $\rho(s) = 1$. In this case, the deviations have infinite persistence and the process has infinite memory, i.e., it never ''forgets''.
3. The assumption of ''homogeneous'' unconditional mean for healthy firms, denoted by $E(X_{i,t}|h) = \mu_h$, is a basic ingredient in every business failure prediction model. The latter assumption is tested in Section 4. The unconditional mean is the long-run equilibrium mean value of the attribute vector $X_{i,t}$. On the other hand, the conditional mean of $X_{i,t}$, denoted by $E(X_{i,t}|I_{t-1}, h) = \mu_{h,t-1}$, is the mean based on the information available up to time $t-1$, or past values of the variables. In the absence of serial correlation $\mu_h = \mu_{h,t-1}$.
4. The CUSUM model can be viewed as an extension of the earlier works of Wecker (1979) and Neftci (1982, 1985) on the prediction of turning points of economic time series. Other relevant contributions in this area include Lorden (1971), Pollak and Siegmund (1975), Siegmund (1985) and Chu and White (1992).
5. The CUSUM score $C_{i,t}$ behaves as a discrete time continuous random process with an upper bound of zero. In the healthy population, the increment (drift) of the process $Z_{i,t} - K$ has a positive mean, provided that $K < D/2$. Thus, $C_{i,t}$ approaches its upper bound of zero with probability one. As the firm's attribute vectors move towards the failed population, $Z_{i,t} - K$ switches to a negative mean and thereafter $C_{i,t}$ accumulates negatively signaling the switch in the distribution of the firm's attribute vector.
6. This sampling approach is similar to that used by Theodossiou et al. (1996).
7. For example, a set of 54 variables will generate 7,590,024 four-variables profiles and 379,501,200 five-variables profiles.
8. Theodossiou and Kahya (1996) examine the behavior of business failure prediction models including explanatory variables similar to those of Altman's (1968) popular Z-score model as well as other popular financial variables; see Appendix II. They show that business failure prediction models based on these variables exhibit deteriorating forecasting performance over time. For example, the error rates of models including Altman's variables start from low levels of about 10% in the late 1970s (1978) and reach levels of more than 30% in the early 1990s (1992). The average error rate for the models are about 17%.
9. A good review of the various methods used in the estimation of the error rates of linear Discriminant analysis and other similar models is given in McLachlan (1992), Chapter 10, pp. 337–377.
10. That is, a larger data set results in statistically more reliable estimates for the model. Also, splitting the data into two or more periods to validate the model over time will result in statistically less reliable estimates for the fitted VAR and CUSUM models.
11. The error rate for failed firms (Type II error) is calculated using the observations corresponding at one year prior to failure. There are 72 such observations. This number does not allow the computation of meaningful annual error rates for failed firms.

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